

## 1 PRG270 Analyse Zufallsrohdaten nach AIS31 (kurz)

```
*****
*
* AIS31 evaluation tests
*
*****
```

date, time: 03/27/2019, 14:21:54  
tested file: PRG270r.rnd  
size of file: 10240000 bytes

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### Introduction

The purpose of the following tests is to evaluate the suitability of a true (physical) random number generator for cryptographic applications. In [1] an evaluation methodology for physical random number generators has been proposed by the German Federal Security Agency. In the mathematical-technical reference to [1], five tests are defined for the P2-evaluation of a physical random number generator (cf. [3] and [4]) which are implemented in the following tests 1 - 5.

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### Results of test 1 (test (P2.i)(vii.a) of AIS 31, cf. [3] and [4])

In this test, the relative frequency  $r$  of bit 1 occurring in the first 100000 bits of the bit sequence is computed. Then the bit sequence passes the test if  $|r - 0.5| < 0.025$ .

test scope: first 100000 bits  
number of ones: 50092  
relative frequency: 0.500920  
test value: 0.00092000 < 0.025

sequence passes test 1

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### Results of test 2 (test (P2.i)(vii.b) of AIS 31, cf. [3] and [4])

In this test, two disjoint sub-sequences  $TF(0)$  and  $TF(1)$  of bit pairs are considered where  $TF(i)$  consists of the first 100000 bit pairs of the form  $(i,x)$  occurring in the bit sequence after the test scope of test 1. Let  $v(i,j)$  denote the relative frequency of all bit pairs of the form  $(i,j)$  in  $TF(i)$ . Then the bit sequence passes the test if  $|v(0,1) + v(1,0) - 1| < 0.02$ .

number of 2-bit words looked up: 200333  
relative frequency  $v(0,1)$ : 0.500600  
relative frequency  $v(1,0)$ : 0.499440  
test value: 0.00040000 < 0.02

sequence passes test 2

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### Results of test 3 (test (P2.i)(vii.c) of AIS 31, cf. [3] and [4])

In this test, 4 disjoint sub-sequences  $TF(0,0), \dots, TF(1,1)$  of 3-tuples are considered where  $TF(i,j)$  consists of the first 100000 3-tuples of bits of the form  $(i,j,x)$  occurring in the bit sequence

after the test scope of test 2. For every  $i, j$  in  $\{0,1\}$ , let  $S(i, j)$  denote the sub-sequence of all bits  $k$  such that  $(i, j, k)$  is element of  $TF(i, j)$ . Then sample  $S(0, j)$  is compared with  $S(1, j)$  for every  $j = 0, 1$ . In this context, a comparison of two bit sequences  $g$  and  $h$  of equal length is performed by a computation of the test value  $t = (g_0 - h_0)^2 / (g_0 + h_0) + (g_1 - h_1)^2 / (g_1 + h_1)$  where  $g_i$  resp.  $h_i$  is the number of bit  $i$  occurring in sequence  $g$  resp.  $h$ . Let  $t_j$  be the test value for the comparison of  $S(0, j)$  with  $S(1, j)$ . Then the bit sequence passes the test if  $t_j < 15,13$  for  $j = 0, 1$ .

number of 3-bit words looked up: 401104  
test value  $t_1$ : 0.737284  $\leq$  15.13  
test value  $t_2$ : 0.016820  $\leq$  15.13

sequence passes test 3

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Results of test 4 (test (P2.i)(vii.d) of AIS 31, cf. [3] and [4])  
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In this test, 8 disjoint sub-sequences  $TF(0,0,0), \dots, TF(1,1,1)$  of 4-tupels are considered where  $TF(i, j, k)$  consists of the first 100000 4-tupels of bits of the form  $(i, j, k, x)$  occurring in the bit sequence after the test scope of test 3. For every  $i, j$  in  $\{0,1\}$ , let  $S(i, j, k)$  denote the sub-sequence of all bits  $b$  such that  $(i, j, k, b)$  is an element of  $TF(i, j, k)$ . Then sample  $S(0, j, k)$  is compared with  $S(1, j, k)$  for every  $j, k$  of  $\{0,1\}$ . In this context, a comparison of two bit sequences  $g$  and  $h$  of equal length is performed by a computation of the test value  $t = (g_0 - h_0)^2 / (g_0 + h_0) + (g_1 - h_1)^2 / (g_1 + h_1)$  where  $g_i$  resp.  $h_i$  is the number of bit  $i$  occurring in sequence  $g$  resp.  $h$ . Let  $t_{jk}$  be the test value for the comparison of  $S(0, j, k)$  with  $S(1, j, k)$ . Then the bit sequence passes the test if  $t_{jk} < 15,13$  for all  $j, k$  of  $\{0,1\}$ .

number of 4-bit words looked up: 806323  
test value  $t_{00}$ : 0.857033  $\leq$  15.13  
test value  $t_{01}$ : 1.705292  $\leq$  15.13  
test value  $t_{10}$ : 4.287390  $\leq$  15.13  
test value  $t_{11}$ : 0.095220  $\leq$  15.13

sequence passes test 4

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Results of test 5 (test (P2.i)(vii.e) of AIS 31, cf. [3] and [4])  
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In this test, the Coron test with the parameters  $L = 8$ ,  $Q = 2560$ , and  $K = 256000$  is performed (cf. [2]). For the first  $Q+K$  8-bit-words after the test scope of test 4, the test value  $f$  of the Coron test is computed. The bit sequence passes the test if  $f > 7.976$ .

8-bit words looked up: 2560 + 256000 bytes  
f-value: 7.99648328  
7.99648328  $>$  7.976

sequence passes test 5

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References  
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[1] AIS 31: Functionality Classes and Evaluation Methodology for Physical Random Number Generators. Version 1 (25.09.2001), (mandatory if a German IT security certificate is applied for; English translation).  
available at [www.bsi.bund.de/zertifiz/zert/interpr/ais31e.pdf](http://www.bsi.bund.de/zertifiz/zert/interpr/ais31e.pdf)

- [2] J.- S. Coron: On the Security of Random Sources. In: Public Key Cryptography - PKC 99. Lecture Notes in Computer Science, Vol. 1560, 29-42, Springer-Verlag, 2002.
- [3] W. Killmann and W. Schindler: A Proposal for: Functionality Classes and Evaluation Methodology for True (Physical) Random Number Generators. Version 3.1 (25.09.2001), mathematical-technical reference of [1] (English Translation); available at [www.bsi.bund.de/zertifiz/zert/interpr/trngk31e.pdf](http://www.bsi.bund.de/zertifiz/zert/interpr/trngk31e.pdf)
- [4] W. Schindler and W. Killmann: Evaluation Criteria for True (Physical) Random Number Generators Used in Cryptographic Applications. In: Cryptographic Hardware and Embedded Systems - CHES 2002. Lecture Notes in Computer Science, Vol. 2523, 431-449, Springer-Verlag, 2002.

## 2 PRG270 Analyse Zufallsrohdaten nach AIS31 (lang)

```
#####
#
#           Results of RawTest
#
#####

date, time: 03/27/2019, 14:22:16
tested file: PRG270r.rnd
size of file: 10240000 bytes

*****
*
*           Results of the frequency test
*
*****
test scope: first 10240000 bytes
relative frequency of bit 1: 0.49885028

block length L = 2: chi^2 = 0.8029, p-value = 0.84877426
block length L = 3: chi^2 = 3.7160, p-value = 0.81184453
block length L = 4: chi^2 = 23.8915, p-value = 0.06695912
block length L = 5: chi^2 = 21.7103, p-value = 0.89190171
block length L = 6: chi^2 = 61.2409, p-value = 0.53928097
block length L = 7: chi^2 = 142.9590, p-value = 0.15776941
block length L = 8: chi^2 = 339.2798, p-value = 0.00031975
block length L = 9: chi^2 = 507.1169, p-value = 0.54018726
block length L = 10: chi^2 = 1019.0559, p-value = 0.52892354

*****
*
*           Results of the serial test
*
*****
test scope: first 10000000 bytes
relative frequency of bit 1: 0.49884703

block length L = 2: chi^2 = 0.3929, p-value = 0.82163189
block length L = 3: chi^2 = 6.8763, p-value = 0.14256942
block length L = 4: chi^2 = 10.6797, p-value = 0.22051571
block length L = 5: chi^2 = 21.3932, p-value = 0.16388572
block length L = 6: chi^2 = 37.7913, p-value = 0.22165605
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block length L = 7: chi^2 = 80.6102, p-value = 0.07849718  
block length L = 8: chi^2 = 137.0836, p-value = 0.27549824  
block length L = 9: chi^2 = 278.1932, p-value = 0.16278436  
block length L = 10: chi^2 = 534.2785, p-value = 0.23974751

```
*****  
*                                     *  
*           Results of the modular monobit test           *  
*                                     *  
*****
```

test scope: first 10240000 bytes  
relative frequency of bit 1: 0.49885028

modular monobit test for block length L = 3:  
bit 0: rf = 0.49889763, chi^2 = 0.24490077, p-value = 0.62068871  
bit 1: rf = 0.49892118, chi^2 = 0.54903998, p-value = 0.45871022  
bit 2: rf = 0.49873207, chi^2 = 1.52637368, p-value = 0.21665751

modular monobit test for block length L = 4:  
bit 0: rf = 0.49873340, chi^2 = 1.11915417, p-value = 0.29010066  
bit 1: rf = 0.49858765, chi^2 = 5.65060635, p-value = 0.01744934  
bit 2: rf = 0.49887124, chi^2 = 0.03598760, p-value = 0.84954109  
bit 3: rf = 0.49920884, chi^2 = 10.53195423, p-value = 0.00117328

modular monobit test for block length L = 5:  
bit 0: rf = 0.49882001, chi^2 = 0.06006282, p-value = 0.80639668  
bit 1: rf = 0.49872534, chi^2 = 1.02300565, p-value = 0.31180712  
bit 2: rf = 0.49890192, chi^2 = 0.17473628, p-value = 0.67593638  
bit 3: rf = 0.49900531, chi^2 = 1.57510598, p-value = 0.20946705  
bit 4: rf = 0.49879883, chi^2 = 0.17349921, p-value = 0.67702049

modular monobit test for block length L = 6:  
bit 0: rf = 0.49884574, chi^2 = 0.00112619, p-value = 0.97322899  
bit 1: rf = 0.49899918, chi^2 = 1.21087189, p-value = 0.27115951  
bit 2: rf = 0.49871808, chi^2 = 0.95450499, p-value = 0.32857524  
bit 3: rf = 0.49894952, chi^2 = 0.53790050, p-value = 0.46330413  
bit 4: rf = 0.49884318, chi^2 = 0.00275658, p-value = 0.95812780  
bit 5: rf = 0.49874606, chi^2 = 0.59324439, p-value = 0.44116727

modular monobit test for block length L = 7:  
bit 0: rf = 0.49882213, chi^2 = 0.03710921, p-value = 0.84724288  
bit 1: rf = 0.49888450, chi^2 = 0.05482455, p-value = 0.81487130  
bit 2: rf = 0.49885767, chi^2 = 0.00255741, p-value = 0.95966747  
bit 3: rf = 0.49892834, chi^2 = 0.28522483, p-value = 0.59329695  
bit 4: rf = 0.49879282, chi^2 = 0.15458067, p-value = 0.69419579  
bit 5: rf = 0.49870984, chi^2 = 0.92322997, p-value = 0.33662832  
bit 6: rf = 0.49895671, chi^2 = 0.53022199, p-value = 0.46651354

modular monobit test for block length L = 8:  
bit 0: rf = 0.49850195, chi^2 = 4.96979083, p-value = 0.02579377  
bit 1: rf = 0.49797471, chi^2 = 31.40130451, p-value = 0.00000002  
bit 2: rf = 0.49829268, chi^2 = 12.73548629, p-value = 0.00035878  
bit 3: rf = 0.49881602, chi^2 = 0.04809137, p-value = 0.82641834  
bit 4: rf = 0.49896484, chi^2 = 0.53758966, p-value = 0.46343337  
bit 5: rf = 0.49920059, chi^2 = 5.02638039, p-value = 0.02496402  
bit 6: rf = 0.49944980, chi^2 = 14.72228713, p-value = 0.00012457  
bit 7: rf = 0.49960166, chi^2 = 23.12495021, p-value = 0.00000152

modular monobit test for block length L = 9:  
bit 0: rf = 0.49876140, chi^2 = 0.28761543, p-value = 0.59175269  
bit 1: rf = 0.49891400, chi^2 = 0.14783269, p-value = 0.70061518  
bit 2: rf = 0.49864934, chi^2 = 1.47008483, p-value = 0.22533231  
bit 3: rf = 0.49903452, chi^2 = 1.23589296, p-value = 0.26626405  
bit 4: rf = 0.49897574, chi^2 = 0.57312173, p-value = 0.44902108  
bit 5: rf = 0.49861473, chi^2 = 2.02006121, p-value = 0.15523288  
bit 6: rf = 0.49889697, chi^2 = 0.07937655, p-value = 0.77814408  
bit 7: rf = 0.49887379, chi^2 = 0.02012525, p-value = 0.88718782  
bit 8: rf = 0.49893213, chi^2 = 0.24390860, p-value = 0.62139726

modular monobit test for block length L = 10:

bit 0: rf = 0.49884607, chi^2 = 0.00058118, p-value = 0.98076672  
bit 1: rf = 0.49877759, chi^2 = 0.17315533, p-value = 0.67732265  
bit 2: rf = 0.49886438, chi^2 = 0.00651383, p-value = 0.93567394  
bit 3: rf = 0.49920068, chi^2 = 4.02334659, p-value = 0.04487459  
bit 4: rf = 0.49881812, chi^2 = 0.03390265, p-value = 0.85391405  
bit 5: rf = 0.49879395, chi^2 = 0.10399579, p-value = 0.74708622  
bit 6: rf = 0.49867310, chi^2 = 1.02874189, p-value = 0.31045434  
bit 7: rf = 0.49893945, chi^2 = 0.26056302, p-value = 0.60973361  
bit 8: rf = 0.49880994, chi^2 = 0.05333537, p-value = 0.81735791  
bit 9: rf = 0.49877954, chi^2 = 0.16397560, p-value = 0.68552235

```
*****
*
*           Results of the autocorrelation test           *
*
*****
```

test scope: first 10000000 bytes  
relative frequency of bit 1: 0.49884703

bit shift d = 1: chi^2 = 0.39278546, p-value = 0.53083887  
bit shift d = 2: chi^2 = 6.48125728, p-value = 0.01090178  
bit shift d = 3: chi^2 = 2.46247860, p-value = 0.11659467  
bit shift d = 4: chi^2 = 8.21160098, p-value = 0.00416234  
bit shift d = 5: chi^2 = 2.05132705, p-value = 0.15207359  
bit shift d = 6: chi^2 = 0.10100523, p-value = 0.75062663  
bit shift d = 7: chi^2 = 5.81991488, p-value = 0.01584571  
bit shift d = 8: chi^2 = 0.35732770, p-value = 0.54999412  
bit shift d = 9: chi^2 = 0.45959177, p-value = 0.49781482  
bit shift d = 10: chi^2 = 10.06823737, p-value = 0.00150847

```
*****
*
*           Results of the dependency test           *
*
*****
```

test scope: first 10240000 bytes  
relative frequency of bit 1: 0.49885028

dependency test for block length L = 3:

bit place 1 if bit 0 = 0:  
rf = 0.49893345, chi^2 = 0.37864621, p-value = 0.53832865  
bit place 1 if bit 0 = 1:  
rf = 0.49890889, chi^2 = 0.18716006, p-value = 0.66529086  
bit place 2 if bit 1 = 0:  
rf = 0.49871916, chi^2 = 0.94101459, p-value = 0.33201705  
bit place 2 if bit 1 = 1:  
rf = 0.49874507, chi^2 = 0.60322352, p-value = 0.43735074  
bit place 3 if bit 2 = 0:  
rf = 0.49880899, chi^2 = 0.09335123, p-value = 0.75995922  
bit place 3 if bit 2 = 1:  
rf = 0.49898669, chi^2 = 1.01361762, p-value = 0.31403773

dependency test for block length L = 4:

bit place 1 if bit 0 = 0:  
rf = 0.49864333, chi^2 = 1.75872734, p-value = 0.18478333  
bit place 1 if bit 0 = 1:  
rf = 0.49853173, chi^2 = 4.14589745, p-value = 0.04173553  
bit place 2 if bit 1 = 0:  
rf = 0.49880659, chi^2 = 0.07839631, p-value = 0.77948256

bit place 2 if bit 1 = 1:  
rf = 0.49893630, chi<sup>2</sup> = 0.30223530, p-value = 0.58248447  
bit place 3 if bit 2 = 0:  
rf = 0.49918787, chi<sup>2</sup> = 4.67858798, p-value = 0.03054084  
bit place 3 if bit 2 = 1:  
rf = 0.49922995, chi<sup>2</sup> = 5.89106667, p-value = 0.01521788  
bit place 4 if bit 3 = 0:  
rf = 0.49864423, chi<sup>2</sup> = 1.74171201, p-value = 0.18692205  
bit place 4 if bit 3 = 1:  
rf = 0.49882280, chi<sup>2</sup> = 0.03089404, p-value = 0.86047698

dependency test for block length L = 5:

bit place 1 if bit 0 = 0:  
rf = 0.49877986, chi<sup>2</sup> = 0.16289789, p-value = 0.68650240  
bit place 1 if bit 0 = 1:  
rf = 0.49867063, chi<sup>2</sup> = 1.05507955, p-value = 0.30434011  
bit place 2 if bit 1 = 0:  
rf = 0.49880547, chi<sup>2</sup> = 0.06595378, p-value = 0.79732184  
bit place 2 if bit 1 = 1:  
rf = 0.49899891, chi<sup>2</sup> = 0.72205166, p-value = 0.39547175  
bit place 3 if bit 2 = 0:  
rf = 0.49892984, chi<sup>2</sup> = 0.20786913, p-value = 0.64844271  
bit place 3 if bit 2 = 1:  
rf = 0.49908117, chi<sup>2</sup> = 1.74307528, p-value = 0.18674964  
bit place 4 if bit 3 = 0:  
rf = 0.49889551, chi<sup>2</sup> = 0.06715974, p-value = 0.79551799  
bit place 4 if bit 3 = 1:  
rf = 0.49870182, chi<sup>2</sup> = 0.72075281, p-value = 0.39589708  
bit place 5 if bit 4 = 0:  
rf = 0.49869205, chi<sup>2</sup> = 0.82243033, p-value = 0.36447055  
bit place 5 if bit 4 = 1:  
rf = 0.49894852, chi<sup>2</sup> = 0.31550860, p-value = 0.57431940

dependency test for block length L = 6:

bit place 1 if bit 0 = 0:  
rf = 0.49902651, chi<sup>2</sup> = 0.85006533, p-value = 0.35653386  
bit place 1 if bit 0 = 1:  
rf = 0.49897180, chi<sup>2</sup> = 0.40228097, p-value = 0.52591361  
bit place 2 if bit 1 = 0:  
rf = 0.49864466, chi<sup>2</sup> = 1.15688457, p-value = 0.28211259  
bit place 2 if bit 1 = 1:  
rf = 0.49879187, chi<sup>2</sup> = 0.09298305, p-value = 0.76041854  
bit place 3 if bit 2 = 0:  
rf = 0.49892653, chi<sup>2</sup> = 0.15917285, p-value = 0.68991920  
bit place 3 if bit 2 = 1:  
rf = 0.49897271, chi<sup>2</sup> = 0.40823654, p-value = 0.52286593  
bit place 4 if bit 3 = 0:  
rf = 0.49884038, chi<sup>2</sup> = 0.00268478, p-value = 0.95867624  
bit place 4 if bit 3 = 1:  
rf = 0.49884606, chi<sup>2</sup> = 0.00048500, p-value = 0.98242984  
bit place 5 if bit 4 = 0:  
rf = 0.49879371, chi<sup>2</sup> = 0.08759486, p-value = 0.76725735  
bit place 5 if bit 4 = 1:

rf = 0.49869826, chi<sup>2</sup> = 0.62962767, p-value = 0.42749192  
bit place 6 if bit 5 = 0:  
rf = 0.49869151, chi<sup>2</sup> = 0.69004807, p-value = 0.40614803  
bit place 6 if bit 5 = 1:  
rf = 0.49900067, chi<sup>2</sup> = 0.61602744, p-value = 0.43252736

dependency test for block length L = 7:

bit place 1 if bit 0 = 0:  
rf = 0.49866151, chi<sup>2</sup> = 0.83597410, p-value = 0.36055072  
bit place 1 if bit 0 = 1:  
rf = 0.49910863, chi<sup>2</sup> = 1.55852617, p-value = 0.21188115  
bit place 2 if bit 1 = 0:  
rf = 0.49898883, chi<sup>2</sup> = 0.45028521, p-value = 0.50219954  
bit place 2 if bit 1 = 1:  
rf = 0.49872602, chi<sup>2</sup> = 0.36062522, p-value = 0.54815921  
bit place 3 if bit 2 = 0:  
rf = 0.49891914, chi<sup>2</sup> = 0.11124742, p-value = 0.73872841  
bit place 3 if bit 2 = 1:  
rf = 0.49893766, chi<sup>2</sup> = 0.17830162, p-value = 0.67283688  
bit place 4 if bit 3 = 0:  
rf = 0.49887474, chi<sup>2</sup> = 0.01403065, p-value = 0.90571028  
bit place 4 if bit 3 = 1:  
rf = 0.49871063, chi<sup>2</sup> = 0.45550608, p-value = 0.49973174  
bit place 5 if bit 4 = 0:  
rf = 0.49864182, chi<sup>2</sup> = 1.01961123, p-value = 0.31261124  
bit place 5 if bit 4 = 1:  
rf = 0.49877829, chi<sup>2</sup> = 0.12101819, p-value = 0.72793280  
bit place 6 if bit 5 = 0:  
rf = 0.49851360, chi<sup>2</sup> = 2.65997786, p-value = 0.10290217  
bit place 6 if bit 5 = 1:  
rf = 0.49940219, chi<sup>2</sup> = 7.11118353, p-value = 0.00766045  
bit place 7 if bit 6 = 0:  
rf = 0.49914447, chi<sup>2</sup> = 2.02997958, p-value = 0.15422268  
bit place 7 if bit 6 = 1:  
rf = 0.49849834, chi<sup>2</sup> = 2.89299085, p-value = 0.08896563

dependency test for block length L = 8:

bit place 1 if bit 0 = 0:  
rf = 0.49801221, chi<sup>2</sup> = 14.42771020, p-value = 0.00014564  
bit place 1 if bit 0 = 1:  
rf = 0.49793708, chi<sup>2</sup> = 17.02789516, p-value = 0.00003683  
bit place 2 if bit 1 = 0:  
rf = 0.49816952, chi<sup>2</sup> = 9.52954792, p-value = 0.00202190  
bit place 2 if bit 1 = 1:  
rf = 0.49841693, chi<sup>2</sup> = 3.83048463, p-value = 0.05032838  
bit place 3 if bit 2 = 0:  
rf = 0.49885372, chi<sup>2</sup> = 0.00024288, p-value = 0.98756586  
bit place 3 if bit 2 = 1:  
rf = 0.49877815, chi<sup>2</sup> = 0.10618502, p-value = 0.74452996  
bit place 4 if bit 3 = 0:  
rf = 0.49908332, chi<sup>2</sup> = 1.11487965, p-value = 0.29102368  
bit place 4 if bit 3 = 1:  
rf = 0.49884590, chi<sup>2</sup> = 0.00039212, p-value = 0.98420140

bit place 5 if bit 4 = 0:  
rf = 0.49927504, chi<sup>2</sup> = 3.70260750, p-value = 0.05432751  
bit place 5 if bit 4 = 1:  
rf = 0.49912592, chi<sup>2</sup> = 1.55284923, p-value = 0.21271531  
bit place 6 if bit 5 = 0:  
rf = 0.49944532, chi<sup>2</sup> = 7.26302691, p-value = 0.00703886  
bit place 6 if bit 5 = 1:  
rf = 0.49945440, chi<sup>2</sup> = 7.46249875, p-value = 0.00629975  
bit place 7 if bit 6 = 0:  
rf = 0.49952289, chi<sup>2</sup> = 9.27541015, p-value = 0.00232251  
bit place 7 if bit 6 = 1:  
rf = 0.49968070, chi<sup>2</sup> = 14.10757558, p-value = 0.00017265  
bit place 8 if bit 7 = 0:  
rf = 0.49820456, chi<sup>2</sup> = 8.54622234, p-value = 0.00346240  
bit place 8 if bit 7 = 1:  
rf = 0.49879973, chi<sup>2</sup> = 0.05229822, p-value = 0.81911140

dependency test for block length L = 9:

bit place 1 if bit 0 = 0:  
rf = 0.49894923, chi<sup>2</sup> = 0.17869412, p-value = 0.67249790  
bit place 1 if bit 0 = 1:  
rf = 0.49887870, chi<sup>2</sup> = 0.01467090, p-value = 0.90359321  
bit place 2 if bit 1 = 0:  
rf = 0.49868570, chi<sup>2</sup> = 0.49415344, p-value = 0.48208038  
bit place 2 if bit 1 = 1:  
rf = 0.49861271, chi<sup>2</sup> = 1.02523509, p-value = 0.31128044  
bit place 3 if bit 2 = 0:  
rf = 0.49890268, chi<sup>2</sup> = 0.05012540, p-value = 0.82284522  
bit place 3 if bit 2 = 1:  
rf = 0.49916696, chi<sup>2</sup> = 1.82076242, p-value = 0.17722255  
bit place 4 if bit 3 = 0:  
rf = 0.49880162, chi<sup>2</sup> = 0.04319242, p-value = 0.83536337  
bit place 4 if bit 3 = 1:  
rf = 0.49915065, chi<sup>2</sup> = 1.63932326, p-value = 0.20041834  
bit place 5 if bit 4 = 0:  
rf = 0.49841123, chi<sup>2</sup> = 3.51646976, p-value = 0.06076174  
bit place 5 if bit 4 = 1:  
rf = 0.49881919, chi<sup>2</sup> = 0.01756392, p-value = 0.89456593  
bit place 6 if bit 5 = 0:  
rf = 0.49860476, chi<sup>2</sup> = 1.10045066, p-value = 0.29416722  
bit place 6 if bit 5 = 1:  
rf = 0.49919092, chi<sup>2</sup> = 2.10654362, p-value = 0.14667027  
bit place 7 if bit 6 = 0:  
rf = 0.49904958, chi<sup>2</sup> = 0.72469579, p-value = 0.39460791  
bit place 7 if bit 6 = 1:  
rf = 0.49869733, chi<sup>2</sup> = 0.42491558, p-value = 0.51449440  
bit place 8 if bit 7 = 0:  
rf = 0.49906048, chi<sup>2</sup> = 0.80613884, p-value = 0.36926427  
bit place 8 if bit 7 = 1:  
rf = 0.49880331, chi<sup>2</sup> = 0.04007269, p-value = 0.84133853  
bit place 9 if bit 8 = 0:  
rf = 0.49891971, chi<sup>2</sup> = 0.08795149, p-value = 0.76679775  
bit place 9 if bit 8 = 1:  
rf = 0.49860230, chi<sup>2</sup> = 1.11708306, p-value = 0.29054742

dependency test for block length L = 10:

bit place 1 if bit 0 = 0:  
rf = 0.49897064, chi<sup>2</sup> = 0.23788177, p-value = 0.62574022  
bit place 1 if bit 0 = 1:  
rf = 0.49858352, chi<sup>2</sup> = 1.16319639, p-value = 0.28080362  
bit place 2 if bit 1 = 0:  
rf = 0.49893230, chi<sup>2</sup> = 0.11048195, p-value = 0.73959610  
bit place 2 if bit 1 = 1:  
rf = 0.49879601, chi<sup>2</sup> = 0.04814479, p-value = 0.82632350  
bit place 3 if bit 2 = 0:  
rf = 0.49902017, chi<sup>2</sup> = 0.47395602, p-value = 0.49117282  
bit place 3 if bit 2 = 1:  
rf = 0.49938214, chi<sup>2</sup> = 4.62413419, p-value = 0.03152517  
bit place 4 if bit 3 = 0:  
rf = 0.49880501, chi<sup>2</sup> = 0.03363095, p-value = 0.85449404  
bit place 4 if bit 3 = 1:  
rf = 0.49883138, chi<sup>2</sup> = 0.00584135, p-value = 0.93907803  
bit place 5 if bit 4 = 0:  
rf = 0.49892235, chi<sup>2</sup> = 0.08529225, p-value = 0.77024964  
bit place 5 if bit 4 = 1:  
rf = 0.49866506, chi<sup>2</sup> = 0.56077159, p-value = 0.45394953  
bit place 6 if bit 5 = 0:  
rf = 0.49858897, chi<sup>2</sup> = 1.12141947, p-value = 0.28961301  
bit place 6 if bit 5 = 1:  
rf = 0.49875775, chi<sup>2</sup> = 0.13995413, p-value = 0.70832662  
bit place 7 if bit 6 = 0:  
rf = 0.49867868, chi<sup>2</sup> = 0.48375578, p-value = 0.48672601  
bit place 7 if bit 6 = 1:  
rf = 0.49920174, chi<sup>2</sup> = 2.01843886, p-value = 0.15539884  
bit place 8 if bit 7 = 0:  
rf = 0.49883950, chi<sup>2</sup> = 0.00190920, p-value = 0.96514799  
bit place 8 if bit 7 = 1:  
rf = 0.49878037, chi<sup>2</sup> = 0.07990295, p-value = 0.77742897  
bit place 9 if bit 8 = 0:  
rf = 0.49898606, chi<sup>2</sup> = 0.30275649, p-value = 0.58215949  
bit place 9 if bit 8 = 1:  
rf = 0.49857216, chi<sup>2</sup> = 1.26428358, p-value = 0.26084182  
bit place 10 if bit 9 = 0:  
rf = 0.49846188, chi<sup>2</sup> = 2.47760626, p-value = 0.11547788  
bit place 10 if bit 9 = 1:  
rf = 0.49923201, chi<sup>2</sup> = 2.38164236, p-value = 0.12276842